Multiscale modeling of sea clutter

Jianbo Gao

PMB Intelligence LLC, West Lafayette, IN 47906

jbgao@pmbintelligence.com,   gao@ece.ufl.edu

Wen-wen Tung

Department of Earth and Atmospheric Sciences

Purdue University, West Lafayette, IN 47907

wwtung@purdue.edu

Jing Hu

PMB Intelligence LLC & Affymetrix, Santa Clara, CA

jhu@pmbintelligence.com

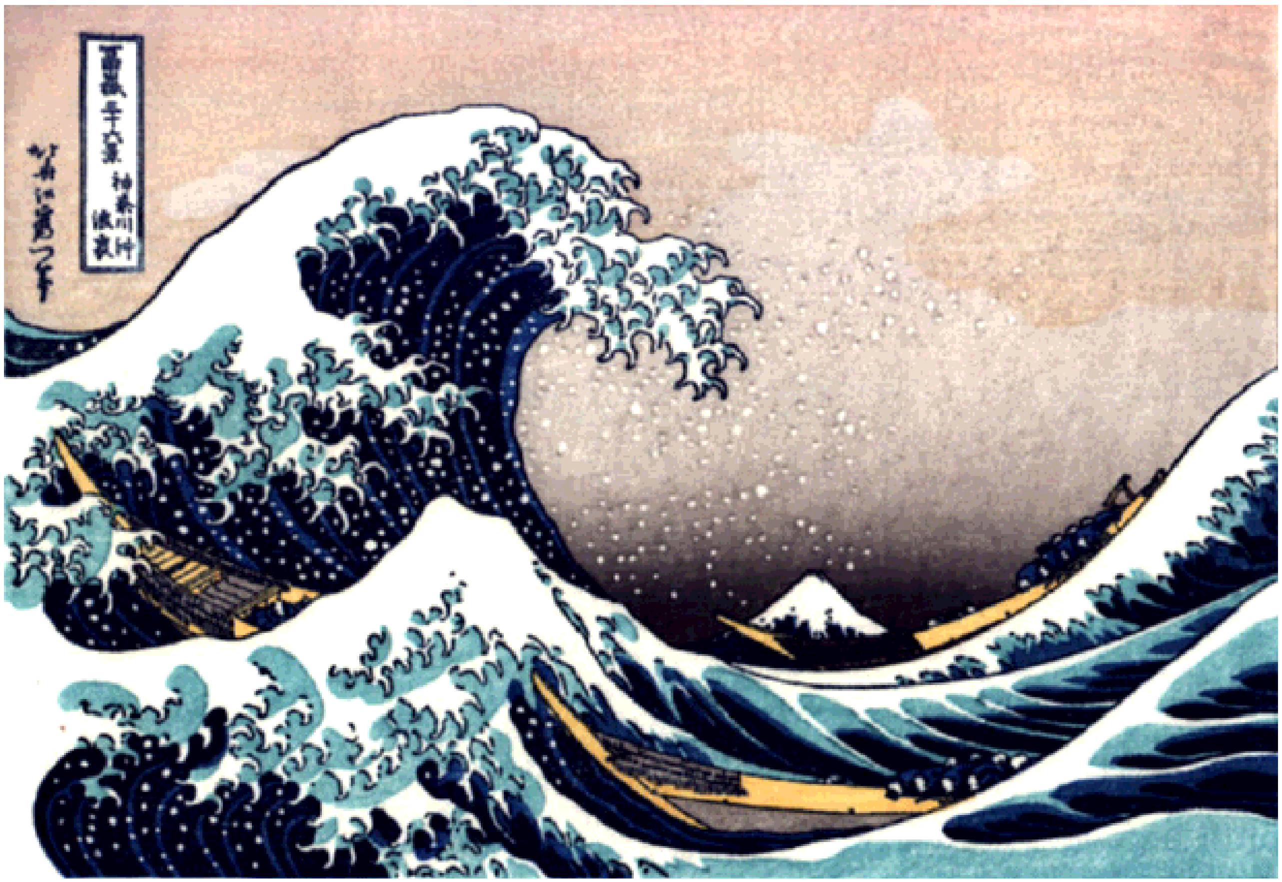
Outline

- Background and challenges
- Fitting nonstationary sea clutter data
- Target detection by fully characterizing the correlation structure of sea clutter
- Target detection by cascade multifractal modeling of sea clutter
- Conclusions

Sea clutter

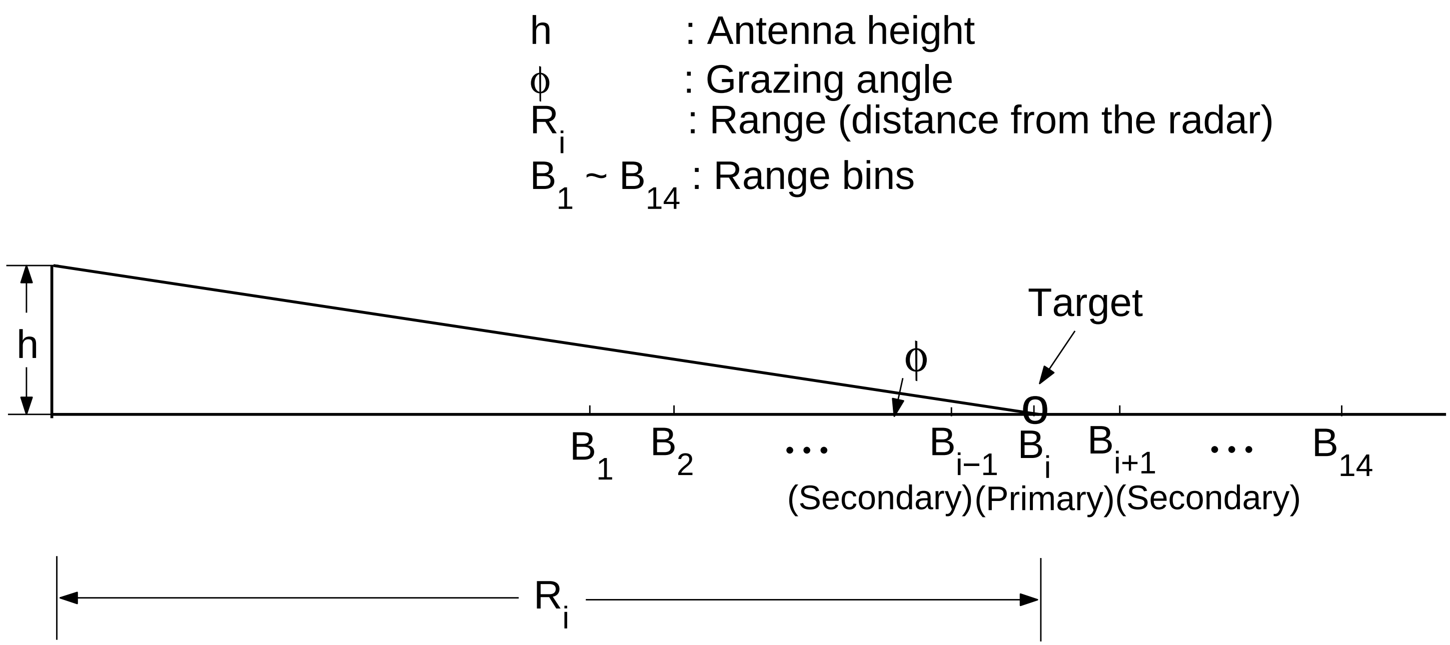
Backscattered returns from a patch of the sea surface
illuminated by a transmitted radar pulse

Complexities: turbulent wave motions + multipath propagation



Source of sea clutter data

- 14 sea clutter measurements from Prof. Simon Haykin; each measurement contains 14 range bins, a few bins hit a small target
- Each measurement was made under certain weather and sea conditions (wave height varied from 0.8 m to 3.8 m; wind conditions varied from still to 60 km/hr)



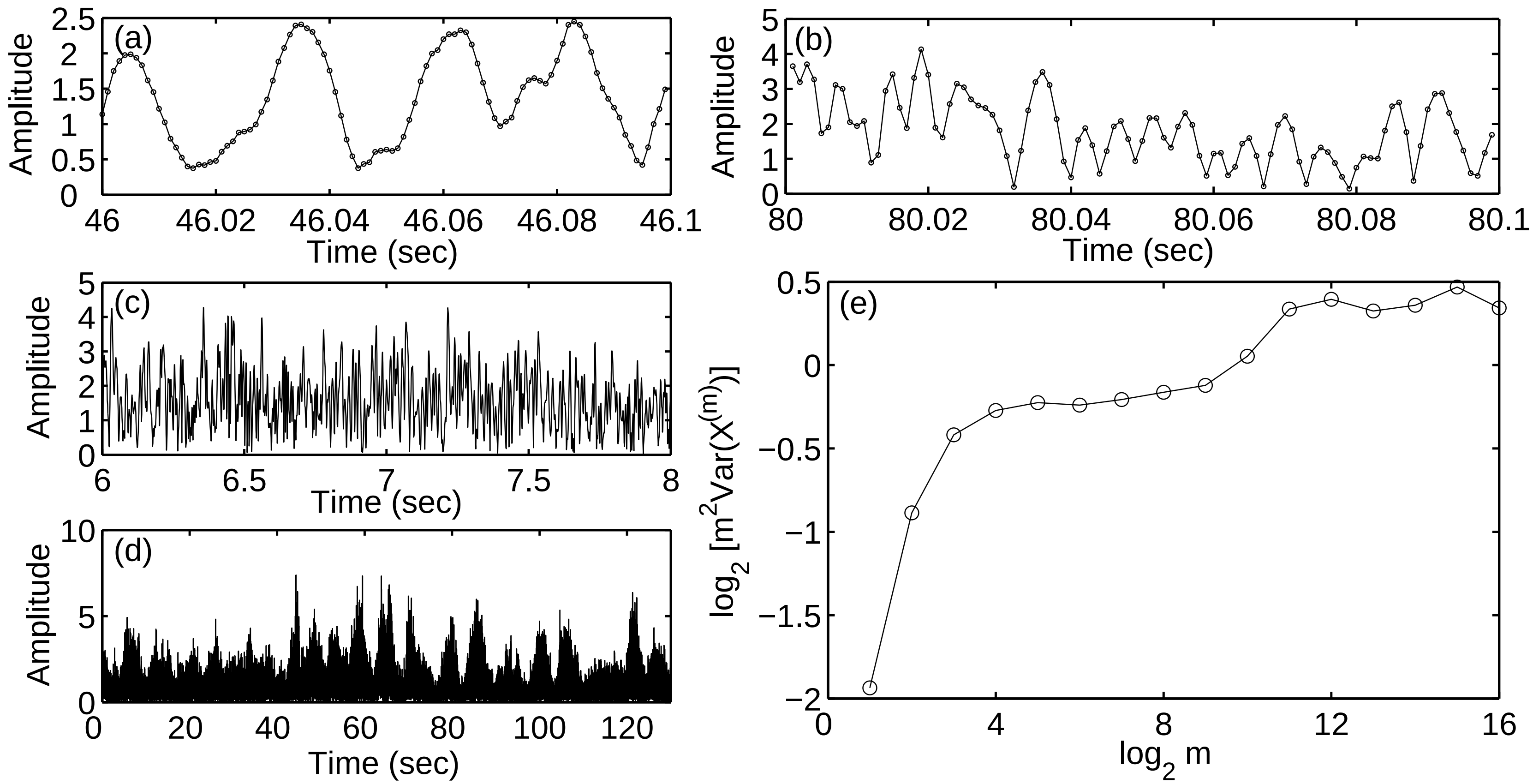
Significance and Challenges of sea clutter modeling

- Sea clutter analysis is an important theoretical problem
- Target detection within sea clutter is important to coastal and national security, to navigation safety, and to environmental monitoring
- **CouldSat**: Sea clutter removal may help improve cloud system modeling
- Over a thousand papers have been published. Numerous methods and new concepts including chaos and fractal theory have been tried to model sea clutter
- By now, the nature of sea clutter is still not well understood
- Simple and effective models for sea clutter are highly desirable

Why sea clutter modeling is difficult? — Nonstationary!

(i) Data viewed at different time and scales appear very different.

(ii) Subplot (e): signals cannot be characterized as ideal random fractals or autoregressive (AR) processes.



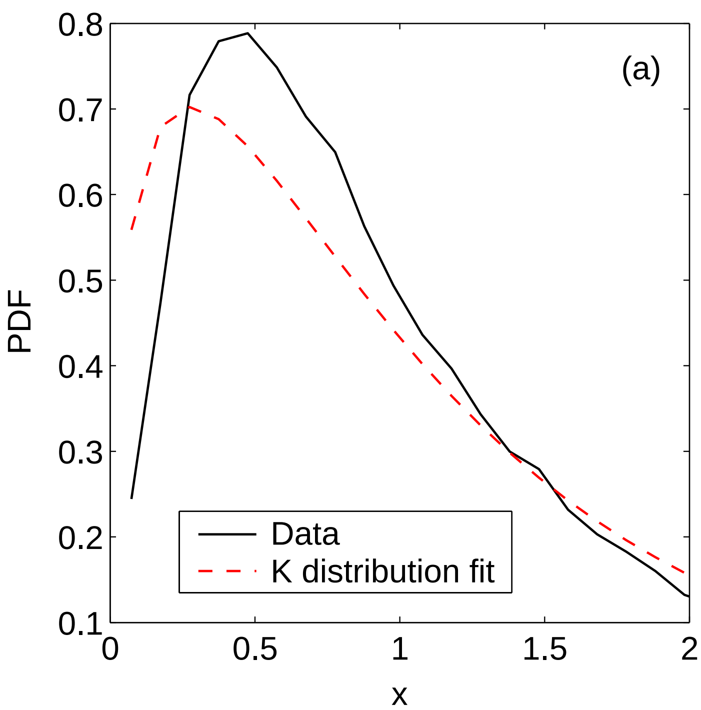
Failure of direct distributional analysis of sea clutter

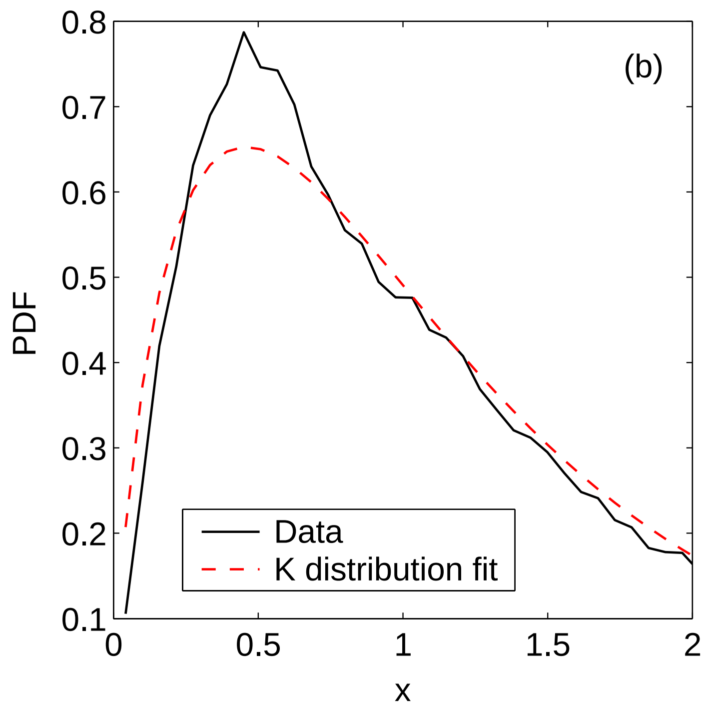
(i) Distr. tried: Weibull, log-normal, K, compound Gaussian, log-Weibull

(ii) K disrt. $f(x) = \frac{\sqrt{2\nu}}{\sqrt{\mu}\Gamma(\nu)2^{\nu-1}} \left(\sqrt{\frac{2\nu}{\mu}}x\right)^{\nu} K_{\nu-1}\left(\sqrt{\frac{2\nu}{\mu}}x\right), \quad x \geq 0$

is among the best; but the fitting can be poor

— Can't help with target detection — Culprit: data is nonstationary!





Our approach to fit nonstationary sea clutter data

- Denote the sea clutter amplitude data by $y(n), n = 1, 2, \cdots$
- Denote the differenced data of sea clutter by,
  $x(n) = y(n+1) - y(n),\ n = 1, 2, \cdots$
- Fit $x(n), n = 1, 2, \cdots$ using **Tsallis distribution**.
- Why such a strategy works?
  - Consider white Gaussian noise, $u(i),\ i = 1, 2, \cdots$. It is **stationary**!
  - Standard Brownian motion (or random walk): $v(n) = \sum_{i=1}^{n} u(i)$ is **nonstationary**, because the variance of $v(n)$ is proportional to (time) n
  - K-distr. can be derived by assuming a random walk model for scatterers!

Tsallis distribution

- Obtained by maximizing the Tsallis entropy under 2 constraints.
- The distr: when $1 < q < 3$,

$$p(x) = \frac{1}{Z_q}[1+\beta(q-1)x^2]^{1/(1-q)},$$

  where Z_q is a normalization constant.
- When $q = 1$ & 2, it reduces to the normal & Cauchy distr.
- When $5/3 < q < 3$, the distribution is heavy-tailed
- Significance: provides foundation for the heavy-tailed and α-stable distr.

Heavy-tailed distribution

- Pareto distr: $P[X \geq x] = \left(\frac{b}{x}\right)^{\alpha}, \ \ x \geq b > 0, \ \ \alpha > 0$
  where α and b are the shape & the location parameters.
- In the discrete time case, we have Zipf distr.
- Heavy-tailed distr: $P[X \geq x] \sim x^{-\alpha}, \quad x \to \infty$
- When $\alpha < 2$, the variance and all higher than 2nd-order moments do not exist.
  when $\alpha \leq 1$, the mean also diverges.
- Cauchy distr (also called Lorentzian distr) with PDF $f(x) = \frac{l}{\pi(l^2+x^2)}$ is an example with $\alpha = 1$

Stable laws and Levy motions

- Paul Levy (teacher of Mandelbrot, the **Father** of fractal geometry) posed such a question: When will the distribution for the sum of the random variables and those being summed have the same functional form?
- Stable laws are the unique class of distributions that have such a property.
- Stable laws include Gaussian distr as a special case; in the non-Gaussian case, the distributions are heavy-tailed
- Levy motions: random walk processes whose increments are characterized by stable laws

The meaning of stable laws and Levy motions

- Normal distr & central limit theorem describe daily, mundane life — Many lucky people live through such a life happily.
- Occasionally one has to take on an unplanned journey, during which many unexpected and exciting (or terrible) things happen.
- Such a journey could be related to hate, love, patriotism, and so on, as illustrated by numerous classic poems, fictions and movies.
- **Kolmogorov was pondering**: Stable laws with infinite variance should be observed more often than the normal distr. In reality ...?
- Abundant examples of heavy-tailed distributions have been found: Amount of Internet traffic, topology of networks (eg, power-law networks), distr. of the size of the power outages, ...
- Fundamental question: How do stable laws arise?

Deriving Tsallis distr by maximizing Tsallis entropy

- Tsallis entropy aims to characterize a type of motion whose complexity is neither regular nor fully chaotic/random, by employing a parameter q, that best describes the motion.
- It's defined by $H_q^T = \frac{1}{q-1}\left(1 - \sum_{i=1}^{m} {p_i}^q\right)$.
- In the continuous case, it is $H_q^T = \frac{1}{q-1}\left(1 - \int_{-\infty}^{\infty} d(\frac{x}{\sigma})[\sigma p(x)]^q\right)$.
- It reduces to the Shannon entropy when $q \to 1$.
- Tsallis distr can be derived by maximizing Tsallis entropy under 2 constraints,
  - Total prob. is 1: $\int_{-\infty}^{\infty} p(x)dx = 1$.
  - Second normalized moment is known: $\int_{-\infty}^{\infty}[x^2 - \sigma^2][p(x)]^q dx = 0$.

Generalized Tsallis distribution

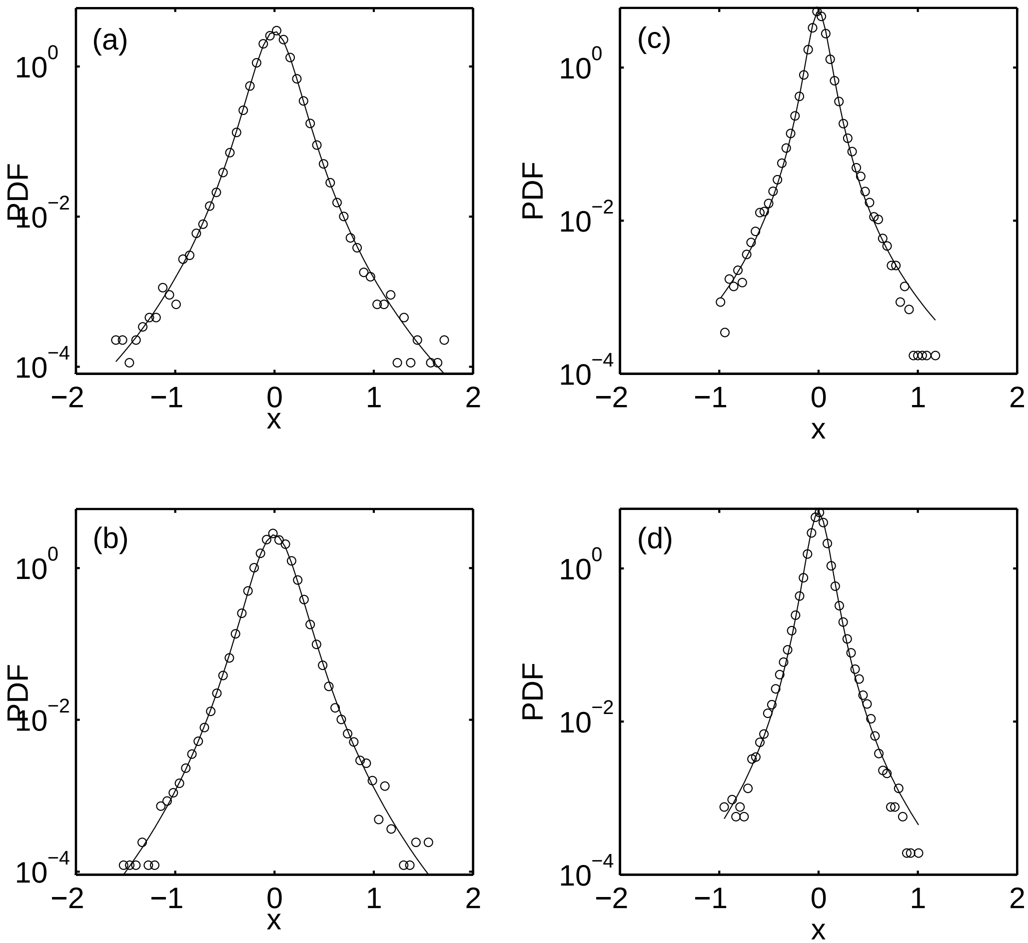
- We may generalize the Tsallis distr by replacing the 2nd constraint by $\int_{-\infty}^{\infty}[x^{\alpha}-\sigma^{\alpha}][p(x)]^{q}dx=0$. Then the distr becomes

$$p(x)=\frac{1}{Z_q}[1+\beta(q-1)x^{\alpha}]^{1/(1-q)}$$

- This is our starting point for modeling sea clutter.
- To model turbulent motions, Christian Beck (2000) obtained the same distr. through a different approach, which is considerably more complicated than our approach.

Fitting sea clutter by Tsallis distribution

(Symbol: data; curve: Tsallis fit)

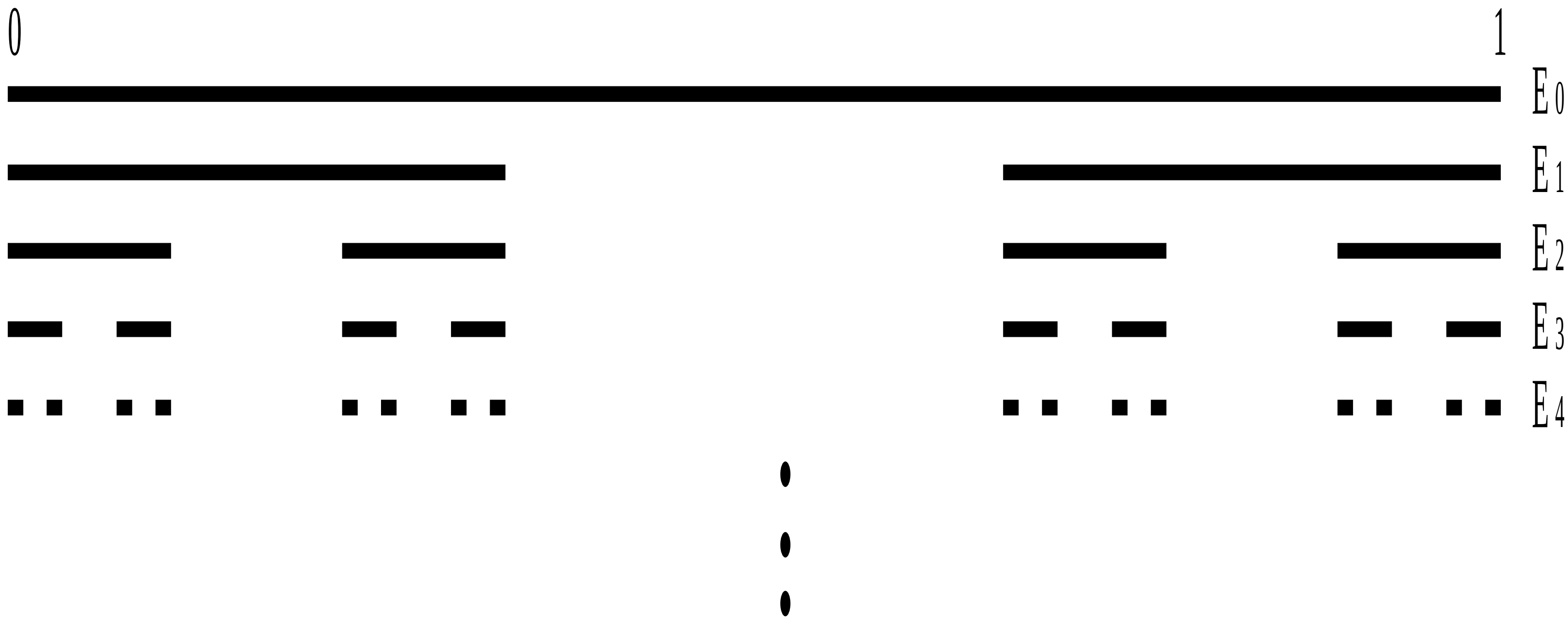


- Data is ready. So what is the challenge here?
- Data is highly nonstationary. It's not very meaningful to perform distributional analysis on original data.
- How about the differenced data? It works!
- Parameters are helpful for target detection.

Introduction to fractal & multifractal

- A part is (exactly or statistically) similar to another part, or to the whole.
- Clouds; mountains; trees; etc. (Images: not computer-made, but photos of Jiu Zhai Gou)
- Power-law relation — a straight line in a log-log plot (scaling)
- Many (or possibly infinitely many) power-law relations — Multifractal.

Cantor set



The set consists of ∞ of isolated points. Its measure and topological dimension are both 0. Fractal dimension $= \ln 2/\ln 3$.

Fractional Brownian motion (fBm) $B_H(t)$

- Gaussian process with mean 0 & stationary increments
- Variance:

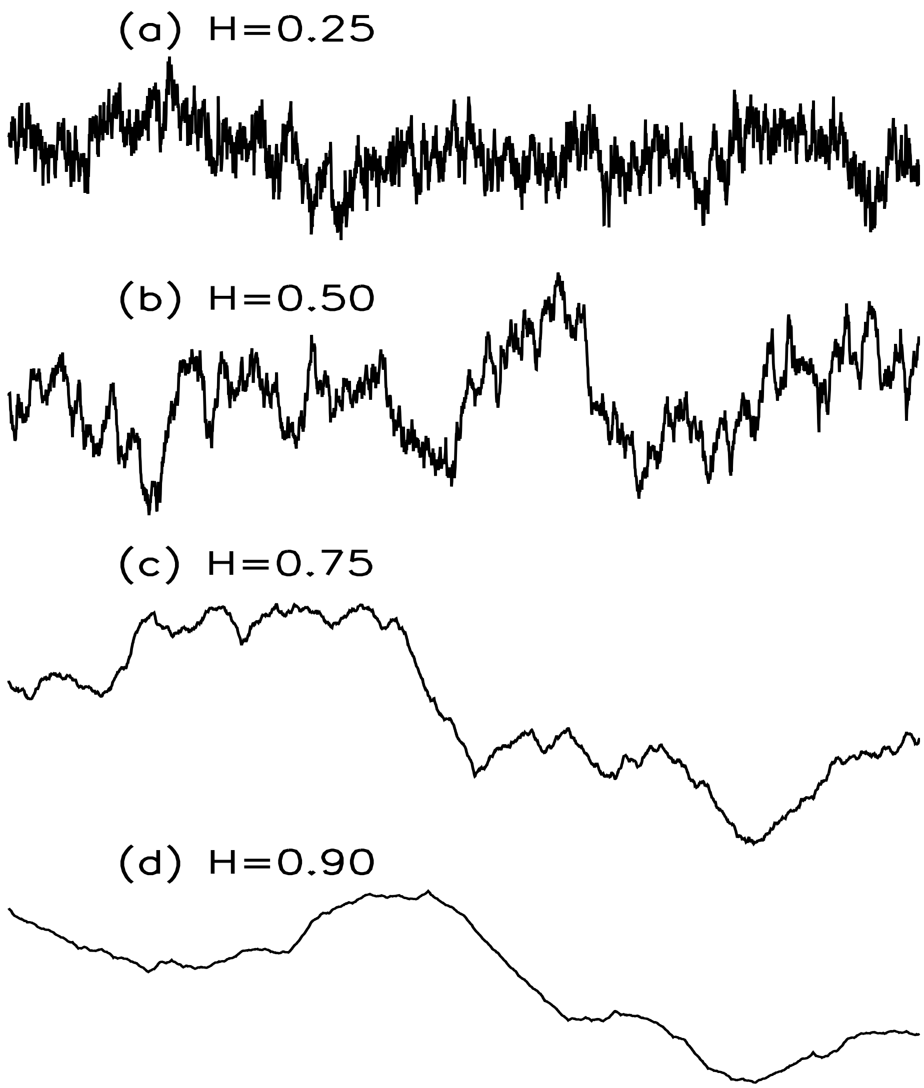
$$E[(B_H(t))^2] = t^{2H}$$

- Power spectral density

$$f^{-(2H+1)}$$

- H: Hurst parameter.
  $1/2 < H < 1$: long memory (long-range-dependence (LRD))
  $H = 1/2$: standard Brownian motion
  $0 < H < 1/2$: anti-persistence
- Applications to a wide range of problems (including Hollywood movie making—fancy landscape)

Examples of fBm processes with different H



Random walks and their analysis

- Remove the mean values from {x(i)} process, denote it as {u(i)}
- Random walk: $y(n) = \sum_{i=1}^{n} u(i)$
- Independent $u(i)$'s (a drunk)–no correlation:
  $E[y(m)^2] = m \cdot E[u(i)^2] \sim m$
- Fluctuation analysis (FA):
  $F^{(2)}(m) = \langle |y(n+m) - y(n)|^2 \rangle \sim m^{\zeta(2)}$
  Hurst parameter $H = H(2) = \zeta(2)/2$
  — $H = 1/2$: no or short-range correlation
  — $0 < H < 1/2$: anti-persistent long range correlation
  — $1/2 < H < 1$: persistent long range correlation

The meaning of the Hurst parameter

- Increment process $\{x_1, x_2, \cdots, x_n\}$: power spectral density (PSD) $f^{-(2H-1)}$; autocorrelation function: $r(k) \sim k^{2H-2}, \;\; as \;\; k \to \infty$
- Random walk process $\{y_n\}$, $y_n = \sum_{i=1}^{n} x_i$, PSD: $f^{-(2H+1)}$
- Averaging the original series X over non-overlapping blocks of size m to obtain:

  $$X_t^{(m)} = (X_{tm-m+1} + \cdots + X_{tm})/m, \;\; t \geq 1, \qquad var(X^{(m)}) = \sigma^2 m^{2H-2}$$

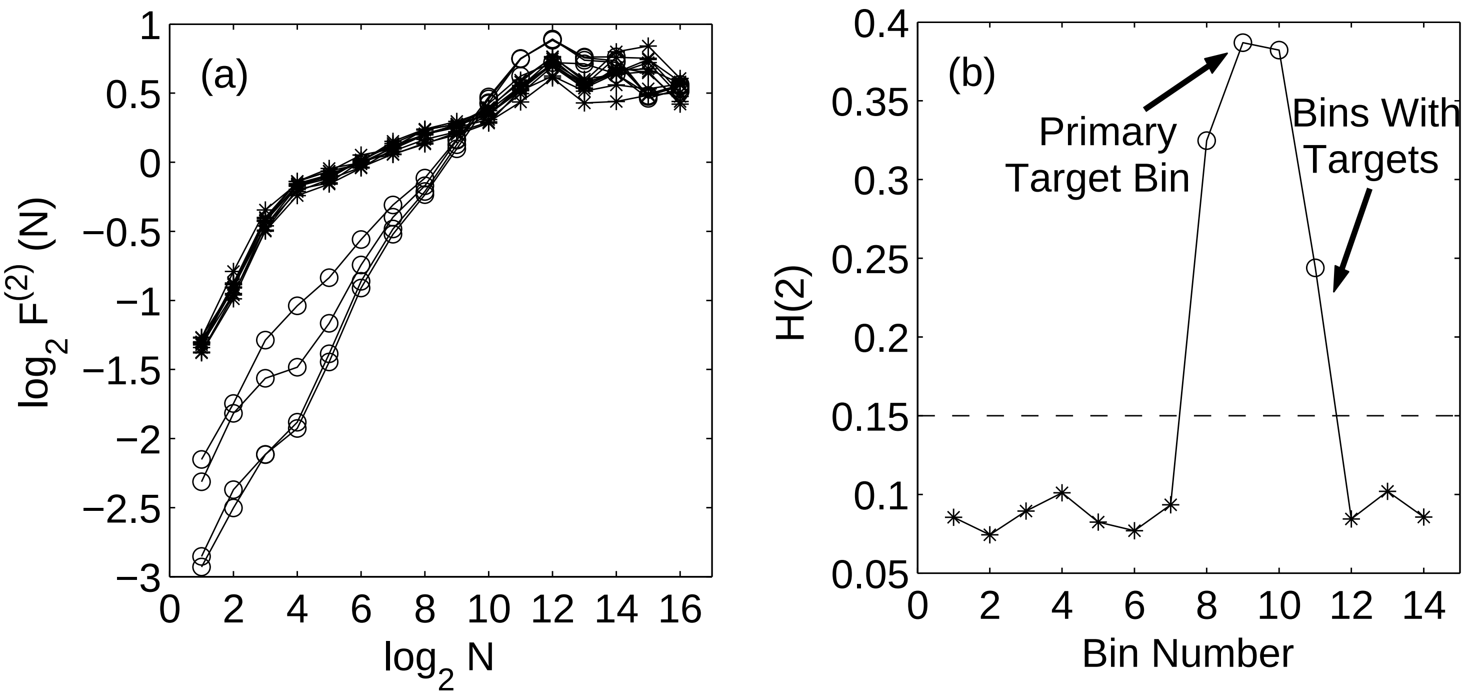
  where σ^2 is the variance of $\{x_1, x_2, \cdots, x_n\}$
- The value of H determines effectiveness of smoothing:
  - $H = 0.50$, $m = 100$, $var(X^{(m)}) = \sigma^2/100$
  - $H = 0.75$, $m = 10^4$, $var(X^{(m)}) = \sigma^2/100$
  - $H = 0.25$, $m \approx 21.5$, $var(X^{(m)}) = \sigma^2/100$

Structure-function–based multifractal analysis

- $F^{(q)}(m) = \langle |y(i+m) - y(i)|^q \rangle \sim m^{\zeta(q)}$?
  $q < 0$: emphasizes small absolute increments of $y(i)$;
  $q > 0$: emphasizes large absolute increments of $y(i)$
- $H(q) = \zeta(q)/q$
- Monofractal: $\zeta(q)$ linear in q **(**$\zeta(0) = 0$**);**
  $H(q)$ constant
  Multifractal: $\zeta(q)$ nonlinear in q;
  $H(q)$ varies with q
- Can extend to detrended multifractal and wavelet-based multifractal analysis
  — **When analyzing real data, these are preferred!** (Gao et al., *Phys. Rev. E* 2006)

Target detection within sea clutter

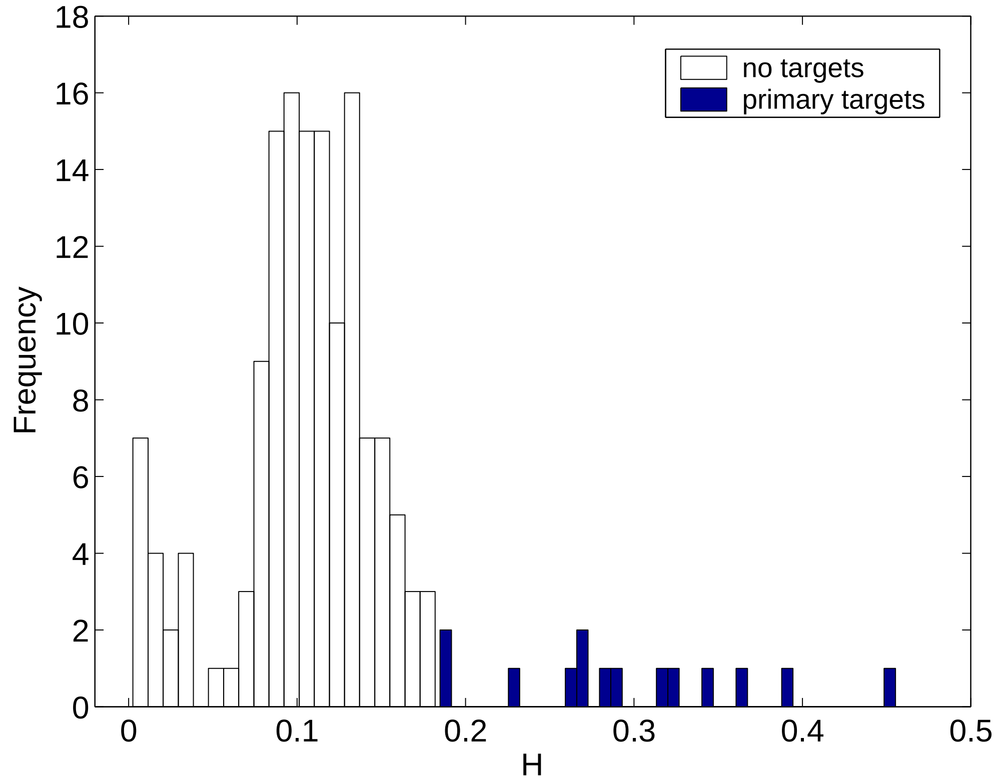
- $H(2)$ is much larger when the range bins hit a target
- Sea clutter data are multifractals, and that other q values can also robustly detect targets within sea clutter



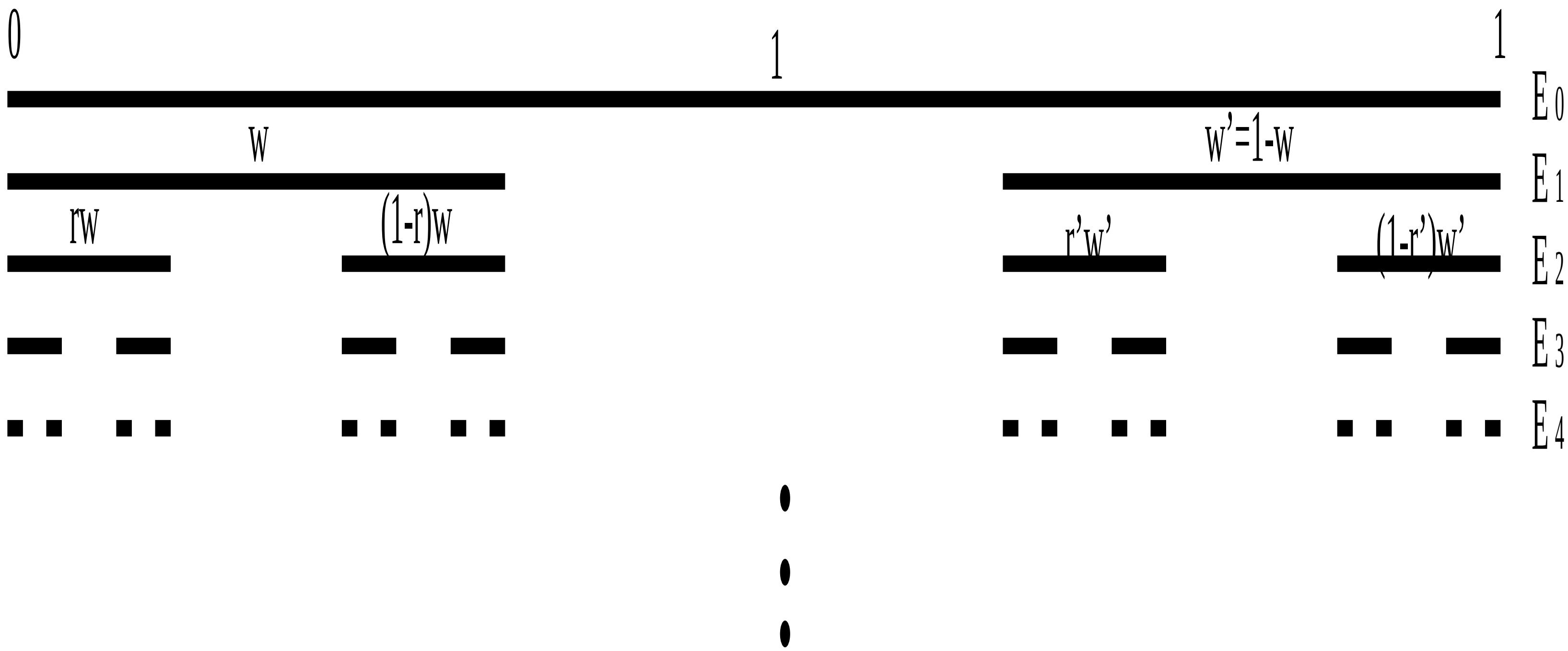
Accuracy of target detection across measurements

- Hypothesis H_0: sea clutter without target, $H(2) < \gamma$

  Hypothesis H_1: sea clutter with target, $H(2) > \gamma$
- $\gamma \approx 0.185$ yields a perfect classification for all datasets

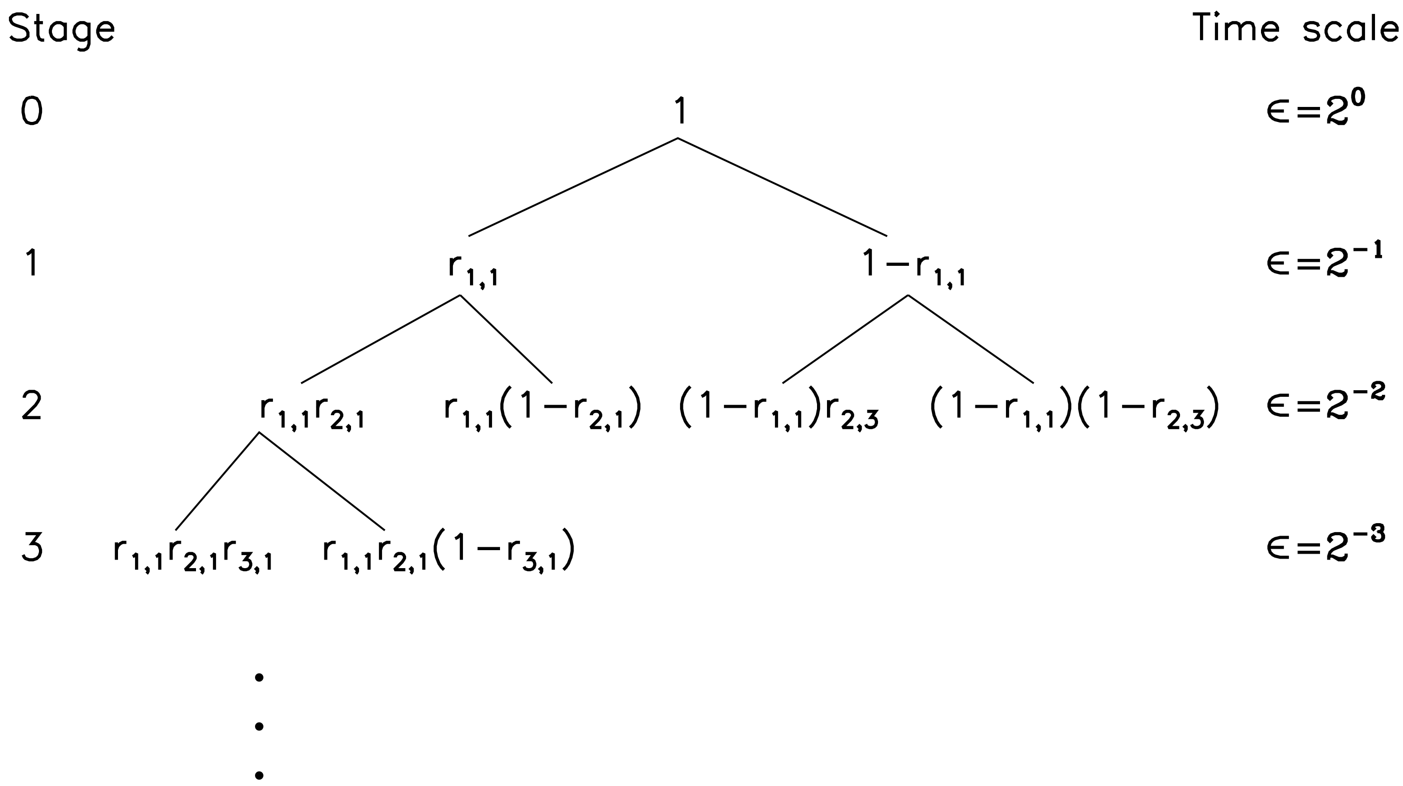


Modeling multifractals: Cantor set with multifractal measure



$w, w', r, r', 1-r, 1-r'$: governed by the same pdf $P(r)$.

Cascade multifractals: construction rule



All $r_{l,m}, 1-r_{l,m}$ are governed by same pdf $P(r)$.

Multifractal scalings for cascade models

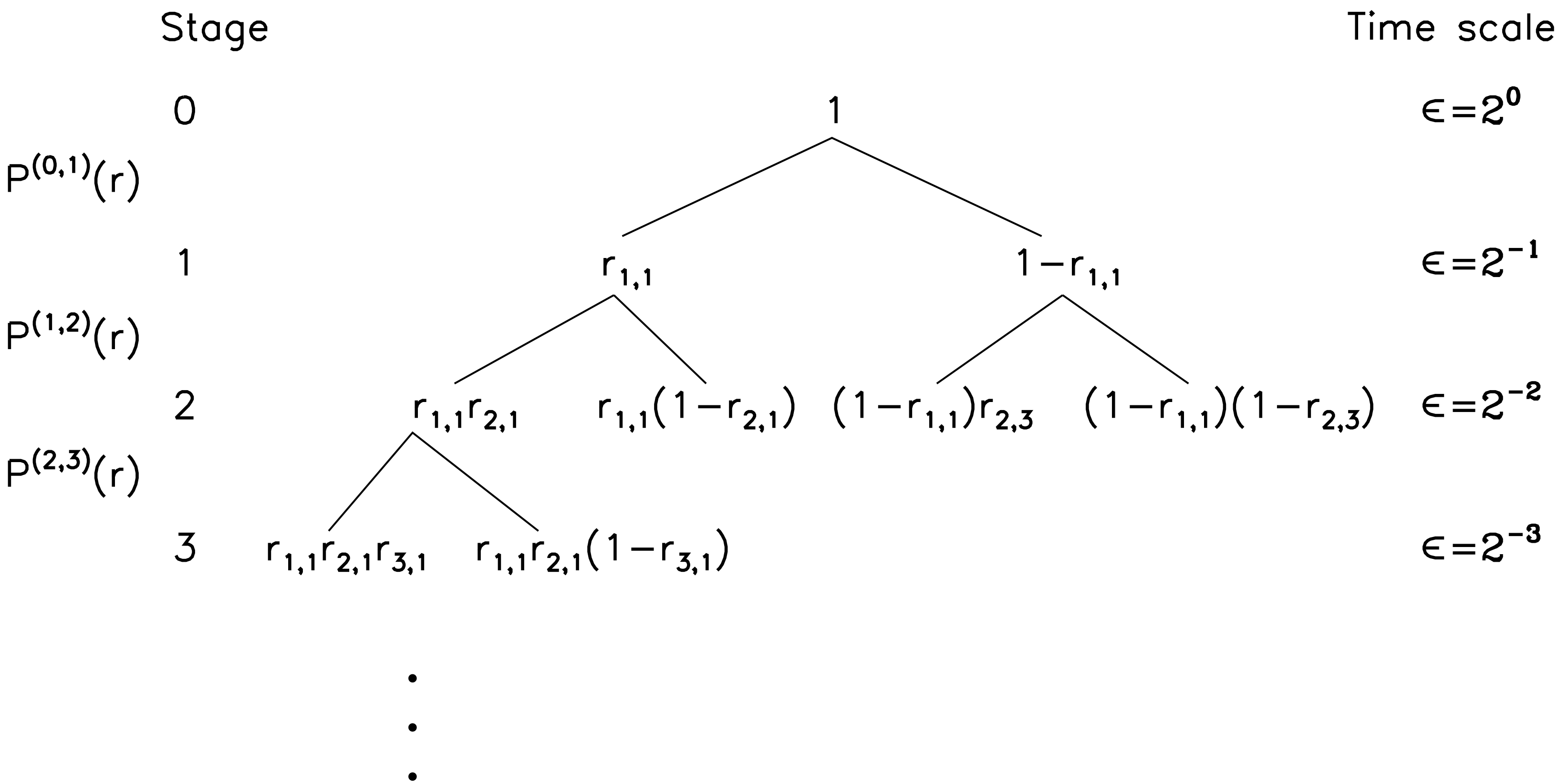
- The weights at the stage N, $\{w_n, n = 1, ..., 2^N\}$, can be expressed as $w_n = u_1 u_2 \cdots u_N$, where u_l, $l = 1, ..., N$, are either r_{ij} or $1 - r_{ij}$.
- Thus, $\{u_i, i \geq 1\}$ are independent identically distributed (iid) random variables (RV's) having pdf $P(r)$.
- Since $\ln w_n$ is the sum of iid RV's $\ln u_i$, $i = 1, ..., N$, one readily sees that $\ln w_n$ follows a normal distribution, and thus w_n follows a log-normal distribution
- Multifractal scaling for the cascade model

$$M_q(\varepsilon) = \sum_i w_i^q \sim \varepsilon^{\tau(q)}, \quad D_q = \tau(q)/(q-1)$$

- We can also prove that

$$\tau(q) = qH(q) - 1$$

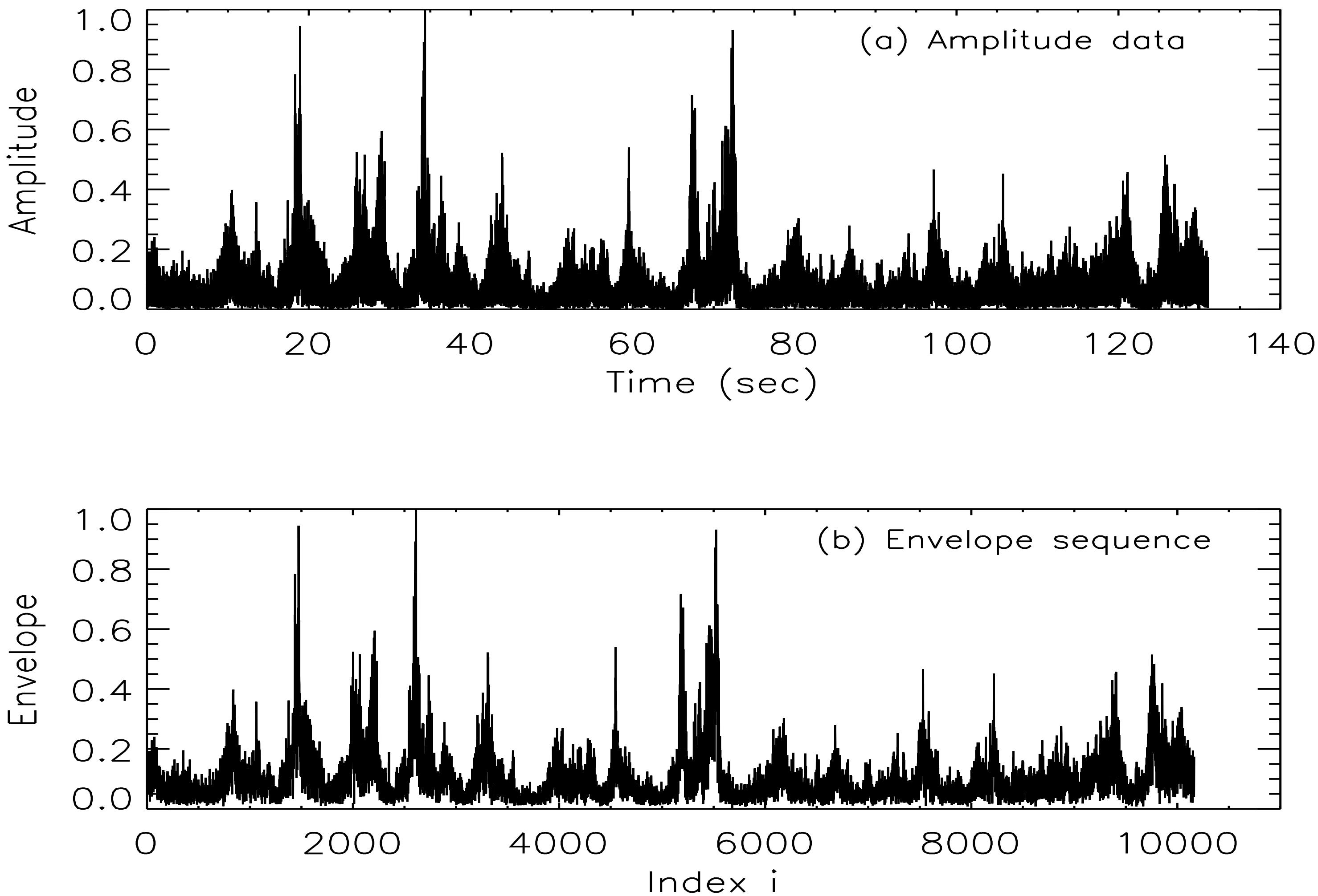
Stage-dependent multiplicative process model



Variance of $P^{(i,i+1)}(r)$ varies from one stage to the next in a simple manner: $\sigma^2_{(i,i+1)} = a \cdot \sigma^2_{(i-1,i)}, \quad a > 1$

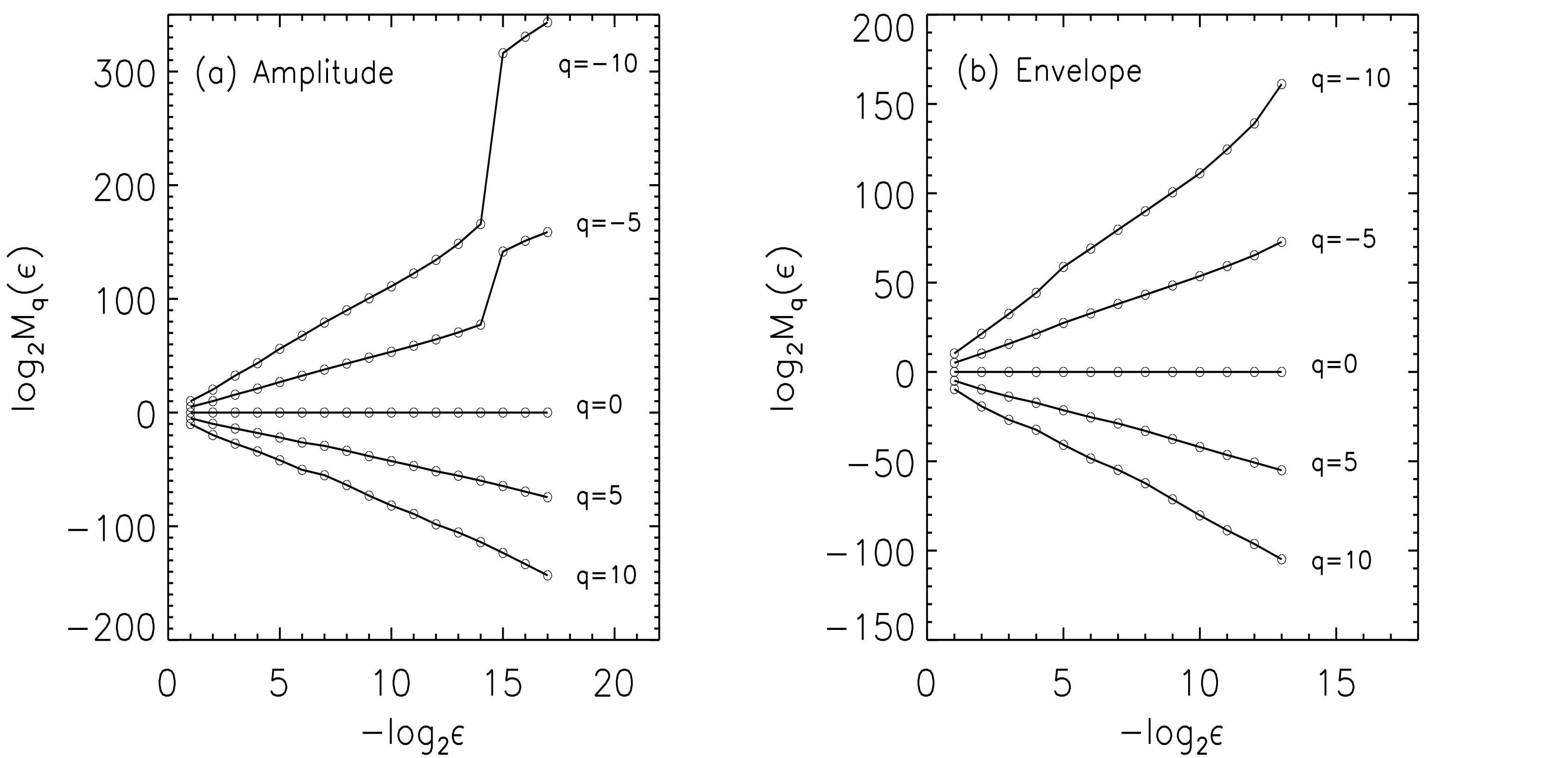
Sea clutter amplitude and envelope data

Envelope is formed by picking up successive local maxima



Multifractal features of sea clutter (Gao & Yao)

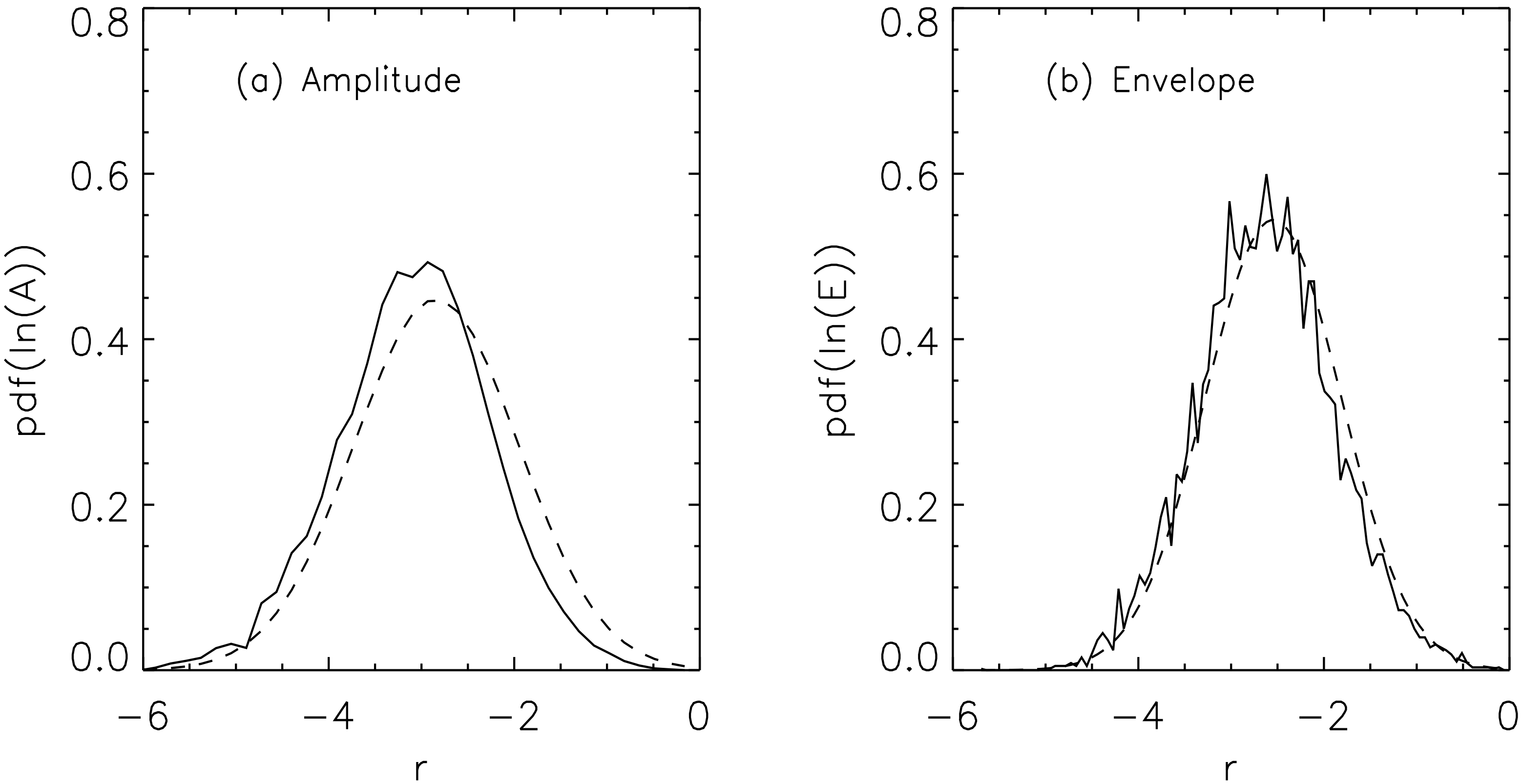
Original signal: scaling breaks for negative *q* and small time scale; indicating the smooth waveform between successive maxima does not follow the multifractal scaling law.



Log-normality of sea clutter envelope signals (Gao & Yao)

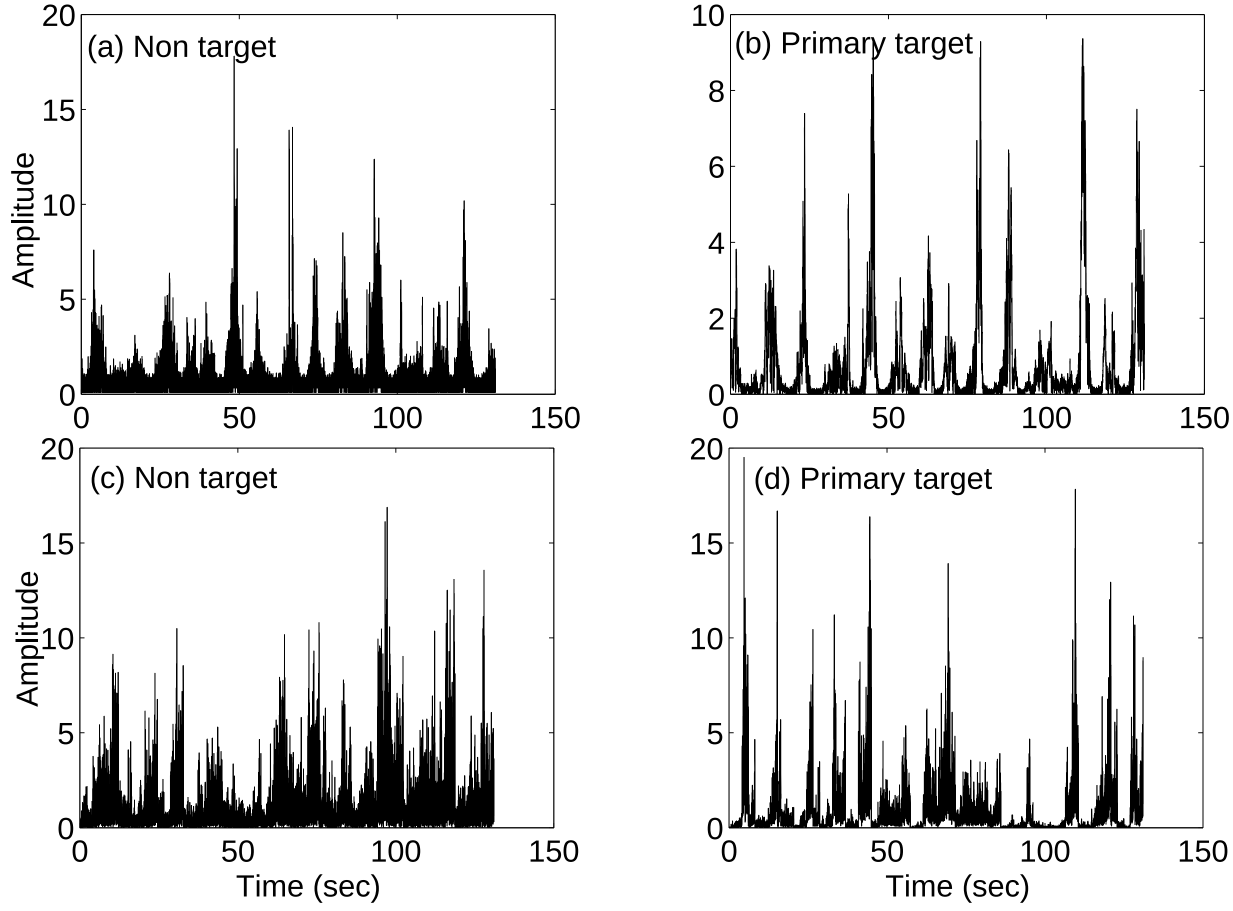
Original signal: slightly deviates from log-normal distribution — due to the smooth waveform part.

Envelope signal: excellent log-normal distribution.

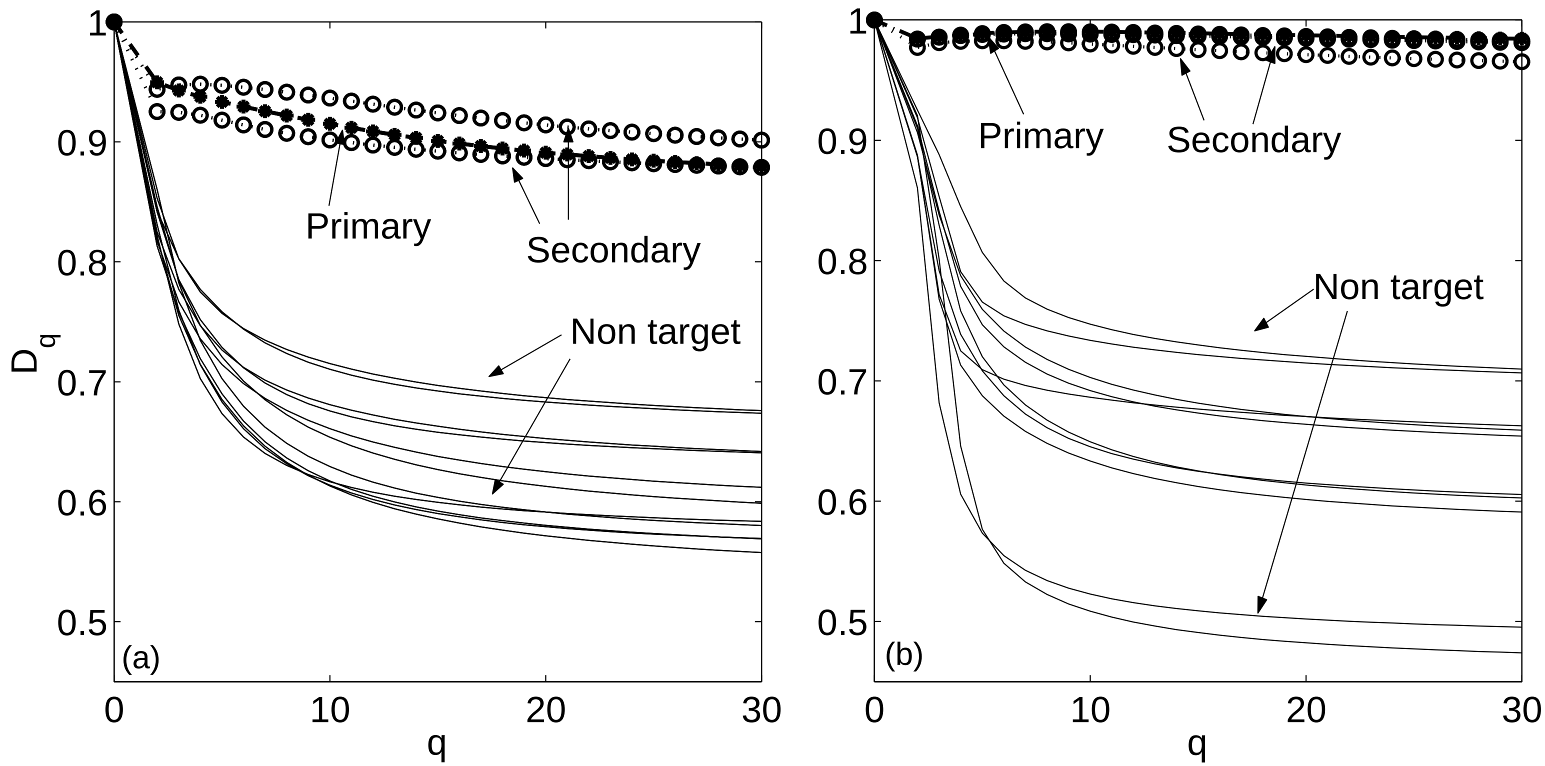


Cascade multifractal modeling of sea clutter

- (a,b) Sea clutter amplitude data without and with target.
  (c,d) The corresponding simulated data.



Target detection by cascade multifractal modeling



Conclusions

- We have shown that sea clutter data are highly nonstationary and multiscaled
- We have developed new distributional analyses approaches to better describe sea clutter
- We have developed structure-function based highly accurate (close to 100%) multifractal methods for detecting low observable targets within sea clutter
- We have developed a cascade multifractal model for sea clutter, which can simultaneously account for the distributional as well as correlation structure of sea clutter
- For more details on the theory, see Gao et al **Multiscale Analysis of Complex Time Series** — *Integration of Chaos and Random Fractal Theory, and Beyond*, Wiley, August, 2007.

Some thoughts on reducing sea clutter from CoudSat data

- Extend the 1-D cascade multifractal model to 2-D and 3-D (after each partition, one square becomes 4 squares, and one cube becomes 8 cubes)
- Identify important spatial scales associated with wave and turbulence patterns on the sea surface; these scales are important elements in multifractal modeling
- Estimate the Hurst parameter (and the $H(q)$ spectrum) from spatial sea clutter data; they may be of critical importance in designing the best spatial smoothing algorithms
- Non-Gaussian sea clutter distribution may also be exploited to improve spatial smoothing